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ABSTRACT
As part of a departmental reform project, a large University converted a "Precalculus College Mathematics" lecture-based course for 97 students into an active learning flipped classroom for first-year engineering students. The curriculum was designed specifically to provide applications of pre-calculus in various engineering subjects. This paper outlines the course evolution, design features, ongoing adjustments made to the course, and quantitative data from the common examination and course evaluations to support the instructional design. The paper concludes with a reflection on which aspects of the learning environment helped facilitate successful learning and guidance for future implementations of a large-scale flipped undergraduate mathematics classroom.

KEYWORDS
Flipped classroom; pre-calculus; active learning; STEM

1. INTRODUCTION

Active learning involves introducing activities in the classroom that actively engage students in the process of learning [7, 16, 27] as opposed to the typical listening and note-taking practices common to the lecture method [13]. Elements of active learning have been around for decades [27]. Active learning approaches have a proven track record of increasing students' confidence, enjoyment, and desire to continue studies in mathematics [4, 10]. It was reported [11] that active learning "has been shown to strengthen student learning and achievement in mathematics, to foster students' confidence in their ability to do mathematics … " (p. 1), which provides further support for the efficacy of active learning. In their meta-analysis of 225 studies, Freeman et al. [13] found that active learning approaches in STEM courses increase students' exam performance by at least half of a letter grade. Relatedly, comparable courses taught through the lecture approach had failure rates that were 55% higher than that of their active learning counterparts [13]. More recently, significant calls for reform in the mathematics education community have led to increased use of active learning approaches in undergraduate mathematics courses [9]. Much of this concern stems from high failure rates in gateway STEM lecture-based courses [29] and many students in traditional lecture settings [8]. In the sections that follow,

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we draw on recommendations from the MAA's Instructional Practices Guide [1] to describe our large-scale active learning course transformation.

1.1. Flipped Classroom Pedagogy

Many mathematics educators have turned to the flipped classroom approach to free up time to afford students more active learning opportunities during the class time [20]. The flipped classroom's primary purpose is to enhance learning through a student-centered approach that incorporates independent learning outside the classroom and collaborative learning inside the classroom [6, 7, 22]. The flipped classroom approach has a long-standing history of being used as an active learning pedagogy, resulting in significant learning gains [21]. Whereas there is no exact definition of a "flipped classroom," many come to understand it as an inversion of instruction and homework [18]. Most contemporary flipped classrooms rely on pre-recorded video instruction outside of class meeting time, followed by some in-class activities facilitated by an instructor [5]. The design of in-person activities and curricular coherence between the at-home and in-person learning environments have been of more recent importance to instructors as they continue to develop successful flipped classrooms [32]. The flipped approach has been very successful in undergraduate STEM courses with multiple studies reporting increases in student achievement [2, 3, 31, 33]. The combination of independent learning outside of class, followed by engaging activities in class with access to an expert and peers, allows students to confront troublesome areas and practice them with support, where they may not have been able to in a traditional lecture setting [6]. Providing students with access to high-quality problems, applications, and help during the class time were the main drivers in choosing a flipped classroom format for this course transformation. Also, we wanted to ensure all students were actively involved in building a solid foundation of important prerequisite content before entering higher levels of proof-based Calculus courses required for the engineering degree.

1.2. Academic Setting

The setting for this pre-calculus course transformation is a large public University in New Jersey with an undergraduate population of approximately 45,000 students. The Mathematics Department at this University started a "Precalculus through Calculus 2" (P2C2) reform project aimed at increasing success and learning by expanding active learning practices across entry-level mathematics courses, including two-semester and one-semester Pre-calculus courses, Calculus 1 and 2 for Life Sciences, and Calculus 1 and 2 for Math and Physical Sciences. This committee comprised tenure track faculty, advisors, part-time lecturers, and department administration and regularly convened to examine data, make programmatic changes, and improve student placement and pathways into future courses. A key program goal was to radically transform mathematics pedagogy across the gateway courses by blending in active learning activities and stronger course coordination among courses with many sections, as they are among the most critical components of a successful P2C2 program [4, 28]. Also, embedding more applications related to students' future studies within their mathematics courses (e.g., environmental examples in Calculus 1 for Life Sciences, engineering applications in Pre-calculus for Engineers, etc.) was an additional priority for the committee. This work required additional outreach to other STEM departments, content from courses in other departments, and time for the instructors to convert interdisciplinary content to the appropriate level problem for their gateway students. This practice of embedding engineering content into gateway courses has been shown to increase students' learning and performance in calculus [24] and pre-calculus [5].

This paper will discuss the development of a large-scale flipped pre-calculus course for first-year engineering students, focusing on the active learning strategies and pre-calculus-based engineering applications that led to positive student outcomes. In addition, the various structures that were implemented to support learning in the flipped classroom will be explained.

2. PRE-CALCULUS FOR ENGINEERS COURSE DESIGN

2.1. Traditional Lecture-Based Pre-Calculus Structure

Precalculus College Mathematics (Math 115) is a one-semester pre-calculus course that meets twice a week in a large lecture setting for 80 min serving approximately 100 students, and once a week for recitation for 80 min serving approximately 33 students each. Course content includes a thorough treatment of algebraic expressions and equations, inequalities, functions, graphing, and function families, including exponential, trigonometric, logarithmic, and rational. The course's adopted textbook is "Pre-Calculus, Mathematics for Calculus" by Stewart, Redlin, and Watson. Students learn approximately two textbook sections per class in the traditional lecture setting, with online homework to complete outside of class. In recitation, students ask questions on their online homework and then take a weekly quiz to assess their knowledge of the previous week's concepts.

2.2. Pre-Calculus for Engineers Pedagogical Technique

As part of our P2C2 reform project, we redesigned Math 115 to make it more of an active learning experience, with an equal focus on students' procedural and conceptual fluency. We created a section strictly for 97 first-year engineering students to begin to build their identities as engineers in an integrated format and have access to parts of the engineering curriculum in the mathematics classroom as early as possible. To bring the engineering curriculum to the mathematics classroom, faculty members from the School of Engineering identified areas where pre-calculus content is part of various engineering concepts. Members of the School of Engineering and the Mathematics P2C2 faculty met to lay out the vision for a flipped class that supported engineering students' understanding of the mathematics necessary for their future courses with time for engineering exploration. A highlight of this collaboration was the development of engineering applications specifically designed for pre-calculus students based on content provided by the School of Engineering. The engineering applications were developed to cover a wide range of engineering fields, such as mechanical, civil, chemical, and electrical, to give students a survey of different applications. Finally, the pre-calculus for Engineers instructor designed course materials, piloted them in a small summer session class of 25 students and implemented them in the Fall 2019 semester. Based on the results of midcourse and end-of-course instructor evaluations, ongoing adjustments were made to the problems themselves and the workflow in lecture and recitation. The flipped classroom design repurposed the same amount of time and class meetings as the traditional version of the course into an active learning experience. Two 80 min lectures turned into two 80 min active learning sessions led by an instructor and four Undergraduate Learning Assistants (ULAs) whose purpose was to facilitate student collaboration during class activities.

2.3. Undergraduate Learning Assistants (ULAs)

ULAs played an important role in the reformed active learning pre-calculus course. ULAs are undergraduate students who apply to work in the classroom alongside the instructor to facilitate group assignments and assist students with understanding the content. ULAs receive formal training in pedagogy through a semester-long pedagogy course offered by the Learning Centers. This 300-level pedagogy course discusses the effectiveness of college teaching methods and instructional strategies needed for peer educators to develop student-centered cooperative learning environments alongside an instructor or graduate teaching assistant. Major course topics include questioning techniques, learning theories, mental models, cooperative learning, metacognition, activity design, and diversity, to name a few. To ensure this training's effectiveness, ULAs also observe actual classes and debrief with the instructional team afterward to better understand how to implement active learning. Once ULAs begin their work in the course they are assigned, they meet weekly with the instructor to plan and debrief on how the classes are going. During these meetings, students apply what they learned in the pedagogy course to actual teaching and learning while working with the instructor to improve class questioning, activities, and overall learning. Research on the use of ULAs to support student success indicates that students have seen dramatic increases in their achievement after working with ULAs [25, 26].

2.4. Pre-Calculus for Engineers Instructional Structure and Facilities

The two weekly lectures were held in a large lecture hall with swivel seating that allowed students to efficiently work with a partner or classmate next to, in front of, or behind them. The class was divided into three smaller groups of approximately 33 students each for recitation once a week for 80 min. Recitations were led by a Graduate Teaching Assistant (GTA) and one ULA. The GTA in this particular course was a Ph.D. candidate who was trained in pedagogy through various workshops and had prior undergraduate teaching experience. The recitation was held in a designated "Active Learning Classroom" with whiteboards and flexible group seating at round tables.

2.5. Flipped Classroom Logistics and Tools Used

Drawing on research from successful flipped classroom designs [15] and active learning in mathematics [19], the initial course structure allowed students to engage with content at home, and content, peers, and the instructional team during class time. First, students were given a video set to watch before coming to class. These videos were typically 5–8 min each, with a typical video set consisting of approximately 6 or 7 videos. The videos were obtained from the Mathispower4u YouTube channel for their clear explanations, visuals, and worked examples. These videos also properly leveraged conceptual and procedural aspects of teaching mathematics, which was important to the instructional team. Students then came to class and engaged in a short five-minute review of main points with the instructor, followed by a one-question clicker quiz for attendance and video-watching accountability. For approximately 40 min, students could individually choose if they wanted to work on select WebAssign problems, engage with dynamic explorations of ideas, or work on new, challenging problems similar to those on exams. The WebAssign problems were online versions of the textbook homework problems given to students in all pre-calculus sections with supporting scaffolds. The dynamic explorations were links to Desmos or GeoGebra activities where students could use sliders to visualize the effects of changing parameters in functions. For example, in one class, students could animate a Ferris wheel on GeoGebra and see how a person's height above ground changed as time passed could be modeled by a trigonometric function. Last, the challenging problems were at the "exam level" and required students to think critically and apply previously learned content to new content. For example, in a lesson on logarithmic functions, a more challenging problem students could choose to work on was to find the inverse of $f(x) = \log_2 x^2$. Once students realized they could not find the inverse of this function since it was not one-to-one, they were challenged to think of a way to modify the function to make it invertible, and what the corresponding new inverse would be.

Students were encouraged to work on the activities of their choice in teams or pairs. The swivel seats and tables in the large lecture hall allowed students to turn in any direction to work with peers easily. During this time, the instructor and four ULAs circulated the room, promoted collaboration, and scaffolded students' questions. To promote collaboration, the teaching team encouraged students to work with a nearby partner, use portable and virtual whiteboards to create a shared problem-solving space, and ask one another questions if a ULA or the instructor was not immediately available. In terms of scaffolding, the teaching team used the reflective toss approach to responding to student questions [30]. With the reflective toss approach, the ULAs were trained to take the meaning of a student's question or statement and throw responsibility for elaboration back to the student. This action leads students to make their thinking and reasoning more visible, which allows the ULA to focus more on the source of difficulty. Another type of scaffolding used by both the teaching team was metacognitive questioning [23]. This was a strategy used to encourage students to explain their understanding and to realize their knowledge limitations. The teaching team especially used connection questions to have students describe how the problem at hand was similar and different from problems they previously solved, strategy questions to have students articulate which strategies were appropriate for solving the problem and why, and reflection questions to have students describe whether the procedure and answers made sense and why. The remaining class time was spent discussing an engineering application that students would later solve in recitation until modifications were made due to mid-course evaluations (addressed in a later section).

2.6. Recitation

This class's recitations were held in an active learning classroom, equipped with wall and personal whiteboards, group seating, and wall-mounted televisions. An active learning classroom layout typically facilitates collaboration and students' ability to make their thinking visible on various surfaces. Students were given a handout of more challenging problems from the previous two classes during recitations and were encouraged to continue working on WebAssign if they did not finish it before recitation. The purpose of the review problems was to give students practice solving problems that combined multiple concepts from multiple class meetings. For example, in a recitation session on solving trigonometric equations, students were given review problems that combined exponential equations with trigonometric equations to expand their thinking. An example of such a problem given during the trigonometry unit is shown in Figure 1. Students also took a weekly recitation quiz and were given time to collaborate on engineering applications, as discussed in greater detail in the next section.

2.7. Engineering Applications in Curriculum Design

In addition to the flipped classroom approach to teaching pre-calculus, the other key component of our course design was the development of a set of engineering applications used to develop students' engineering interests while exposing them to concrete examples of where mathematics can be found in various fields of engineering. Faculty from various engineering departments sent over materials from their courses, which were reviewed by mathematics department faculty, and further refined and developed into exercises appropriate for a pre-calculus audience. An example from the chemical engineering curriculum is shown in Figure 2.

Math 115
7.4-7.5 Review Questions

2. Let $f(x) = p^x$ and $g(x) = p^{cos^2 x}$.

a. Evaluate $f(cos(3\pi))$.

b. Solve $p = p^{tan^2 x}$ exactly on the interval $[0, 2\pi)$.

c. Let $h(x) = log_p(\frac{p}{g(x)})$. Express $h(x)$ as a simplified single trigonometric function. (Hint: consider the laws of exponents).

Figure 1. Sample recitation review problem.

Example from Chemical Engineering Kinetics

A chemical reactor called a continuously stirred tank reactor (CSTR) is used to convert a reactant A into a product P.

A → P

Not all of the reactant will react, and the fraction that reacts is referred to as the fractional conversion *X*. The flowrate of A entering the reactor is F_{A0}, the volume of the reactor is *V*, and the rate of the chemical reaction is *r*.

Their relationship is given by $V = \frac{F_{AO}X}{-r}$. (1)

The reaction is second order in the concentration of A, which means that the rate of reaction is proportional to the square of the concentration, with a proportionality constant k. The negative sign indicates that A is consumed in the reaction.

This relationship is given by $r = -kC_A^{\ 2}$. (2)

The relationship between the concentration of $A(C_A)$ and the conversion (*X*) is given by $C_A = C_{A0}(1 - X)$ where C_{A0} is the concentration of A entering the reactor. (3)

Substituting equations (2) and (3) into (1), we obtain:

$$V = \frac{F_{AO}X}{kC^2_{A0}(1-X)^2} \qquad (4)$$

If F_{A0}, C_{A0}, *V*, and *k* are known, we can solve for the expected conversion in the reactor (X) by rearranging equation 4 into a quadratic equation. Solve for X in terms of these parameters.

Figure 2. Chemical engineering application.

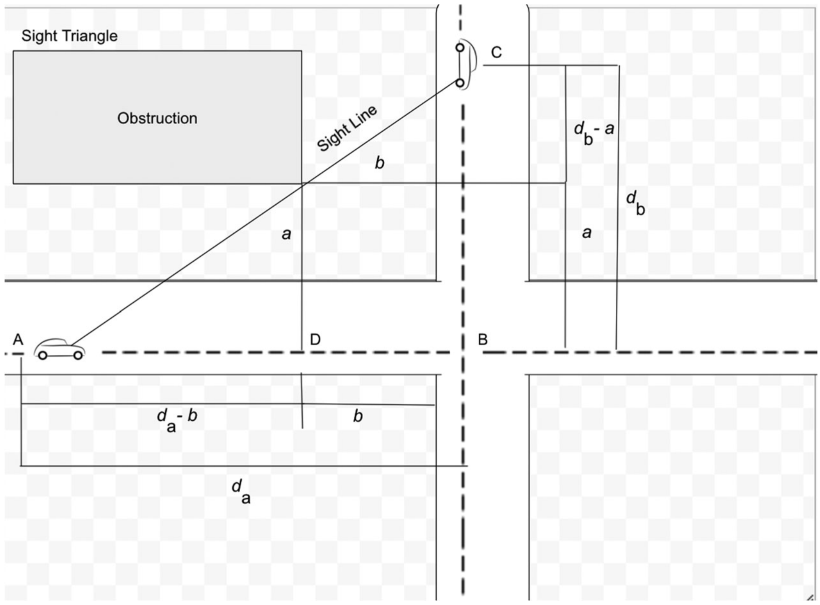


Figure 3. Traffic intersection engineering application (adapted from [14]).

This problem has students explore chemical reactions while learning how to set up equations to model relationships in chemistry. What makes it challenging is that students are asked to apply their knowledge of the quadratic formula to solve a rather complicated quadratic equation, where one of the roots is extraneous in the context of the problem. Pointing this out and discussing why one root is extraneous gets students to think about reactants, products, and conversion factors in a new way.

Another example of one of our more popular engineering applications is the "Designing a Drawbridge" problem. The premise of the problem is that whereas no cars should ever be on a drawbridge when it goes up, things happen, and engineers need to be reasonably sure a car cannot slide off of the opened bridge. Given various trigonometric equations, students are asked to create a function to determine the force of friction for any car on the drawbridge for a given set of bridge angles and friction coefficients. Then, they have to test their equation using a variety of materials for both wet and dry surfaces. This engineering application truly has students think like engineers while also strengthening their fluency with trigonometric equations.

We engaged students in conversations about traffic flow and designing intersections when discussing similarities and proportions early in the course. The engineering application in Figure 3 shows a problem where students were challenged to determine relevant speed limits, minimum sight distance needed for imminent collision, and roadway distances.

For some of the more challenging pre-calculus topics like double and half-angle formulas, we have students work on problems where they have to manipulate formulas to show how they can be rewritten in other equivalent ways. We also

In engineering mechanics, the stress transformation equations for in-plane normal and shear stresses are derived using equilibrium of forces along axes x' and y'. The equations are:

$$(1)\quad \sigma_x{}' = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau(2\sin\theta\cos\theta)$$
$$(2)\quad \sigma_y{}' = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - \tau(2\sin\theta\cos\theta)$$
$$(3)\quad \tau' = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau(\cos^2\theta - \sin^2\theta)$$

Show that equations (1), (2), and (3) can be written as:

$$(1)\quad \sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau \sin 2\theta$$
$$(2)\quad \sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau \sin 2\theta$$
$$(3)\quad \tau' = -\frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta + \tau\cos 2\theta$$

Figure 4. Engineering application on stress transformations.

discuss multiple representations and approximations in engineering to bolster the importance of the problem. An example problem from engineering mechanics is shown in Figure 4.

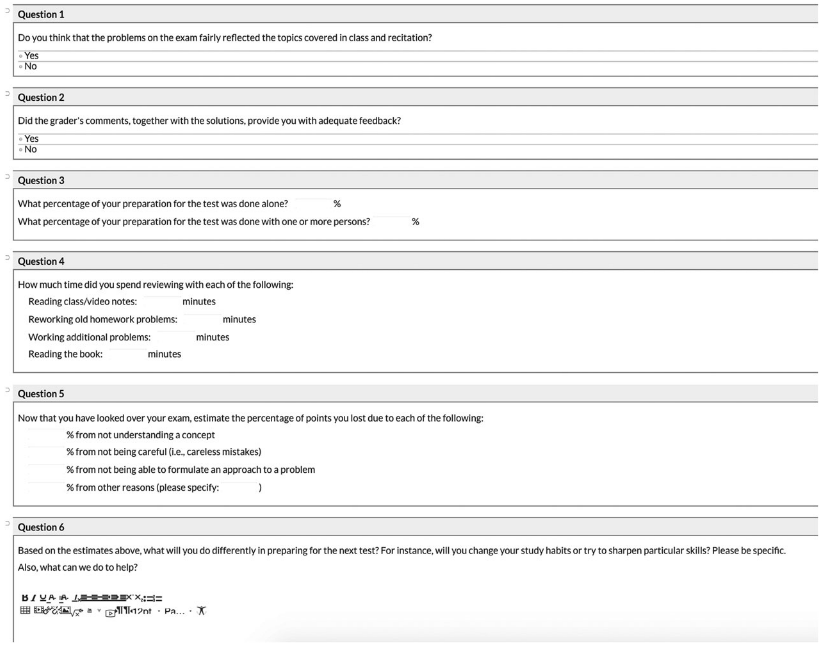
Students had approximately 30 min to work on these problems in small groups under the teaching team's facilitation in a typical recitation. Since the recitation classroom had round tables, there was greater ease of collaboration among students. The GTA reviewed the solutions to these problems with students upon completion.

2.8. Additional Active Learning Strategies

The aforementioned problems, group collaboration, and flipped learning activities were key active learning activities in our setup. We also turned assessment into a more active and ongoing process with opportunities for metacognitive reflection. In addition to the one-question quizzes in the lecture, all recitations ended with a pencil-and-paper quiz, similar to those given in traditional lecture-based recitations. These quizzes were more summative compared to the one-question quizzes in the lecture. In recitations, students also completed exam wrappers [12, 21] after each midterm to reflect on their preparation, the course itself, and future goals. Students completed the exam wrappers on the learning management system, and the instructor and GTA reviewed them, conferenced with individual students, and made necessary course changes as needed. Figure 5 gives an example of the exam wrapper used in Math 115 after the first exam. This metacognitive activity added an additional layer of active learning to the course.

2.9. Course Coordination

During the same semester in which this flipped class was run, eight additional sections of the course ran as traditional lectures. These lectures covered the same content as the flipped course did and took similar midterms and the same final


Question 1
Do you think that the problems on the exam fairly reflected the topics covered in class and recitation?
Yes
No

Question 2
Did the grader's comments, together with the solutions, provide you with adequate feedback?
Yes
No

Question 3
What percentage of your preparation for the test was done alone? %
What percentage of your preparation for the test was done with one or more persons? %

Question 4
How much time did you spend reviewing with each of the following:
Reading class/video notes: minutes
Reworking old homework problems: minutes
Working additional problems: minutes
Reading the book: minutes

Question 5
Now that you have looked over your exam, estimate the percentage of points you lost due to each of the following:
% from not understanding a concept
% from not being careful (i.e., careless mistakes)
% from not being able to formulate an approach to a problem
% from other reasons (please specify:)

Question 6
Based on the estimates above, what will you do differently in preparing for the next test? For instance, will you change your study habits or try to sharpen particular skills? Please be specific.
Also, what can we do to help?

Figure 5. Exam wrapper administered after the first exam.

examination, but students were responsible for completing the same WebAssign problems outside of class. The lectures also met twice a week for 80 min each and one 80-min recitation per week. During the traditional recitation, the students had the opportunity to ask questions about their WebAssign problems. Students took pencil/paper quizzes in the traditional recitations as well. The midterm exams were somewhat coordinated using guidance from the course coordinator. Each instructor had the autonomy to write his or her own non-cumulative, 80-minute exam, and the coordinator looked at all of them and provided feedback to ensure all exams were of comparable difficulty. The final exam was a common, cumulative, three-hour exam for all students in all sections of Math 115. Students were given partial credit according to a rubric for progress towards a correct solution on all exams. Most questions on all exams were open-ended with partial credit and a handful of true/false or "select all that apply" questions. All exams had two parts: half calculator, half non-calculator.

2.10. Changes Due to Mid-Course Evaluation

The mid-course evaluations and post-exam #1 reflections indicated that most students (90%+) had positive things to say about the flipped classroom setup. The students who had negative feelings were conferenced with individually so that the instructional team could help them be more successful in the course. In conferencing with the students, the sources of their negative feelings came from previous negative flipped classroom experiences in high school, difficulty keeping up with the workload, and trouble maintaining focus. This information allowed the instructor to intervene and work with the students to improve their approach to learning in a flipped classroom. Of the recommendations made on both evaluations, the two most common were (1) more teaching from the instructor and (2) more work in recitation. As a result, the engineering applications were completely moved to recitation, where students could have more time to explore the problems in a smaller group setting. The instructor then used the last 20 min of the lecture time to review "exam-type" problems or play a review game in whole-class format, such as a Desmos Card Sort, Quizizz game, or Quizlet Live matching competition. All class sessions ended with a short one-question quiz to hold students accountable for staying in class and completing assignments.

3. EFFECTIVE PRACTICES

A few practices seemed to stand out in reflecting on why the flipped classroom setup was positively received by students and resulted in more significant learning gains than students' gains in a lecture-based setting. The subsections below outline practices that we felt contributed to our students' success and would recommend them to faculty at similar institutions who want to offer a large-scale flipped course for a specific audience. We note, where applicable, course evaluation data and students' final exam performance data to support our claims.

3.1. Student Choice

Flipping the classroom and allowing students some "choice" during the lecture helped them learn how to navigate the complexities of learning larger volumes of material in the college setting. Some students wanted to work on the more challenging problems in class, whereas others needed more remediation from the instructional team. Since we did not force everyone to work on the same thing at the same time, this was possible and highly effective. Students were forced to prioritize what they needed the most help on and get that while in the classroom. Students who were comfortable with procedures spent more time on conceptual problems in class, and students who struggled with learning the targeted concepts from the videos required additional support learning procedures during class time. The degree of personalization allowed in learning in the flipped setting is an area we continue to experiment with.

3.2. Aligning Assessment and Instruction

Having students work through released exam questions as they learned each topic was a practice we encouraged as we wanted them to do the same thing in their other classes. Many students typically postpone exam review until a few days before the exam; in our course, by flipping the classroom, we could encourage early and frequent exam preparation during class time under the instructional team's

Table 1. Common final exam data by section.

Section	Mean	*N*	Std. Deviation
1	70.8012	82	18.64916
2	66.9224	85	22.19806
3	68.8765	81	21.07984
4	65.2208	77	26.38512
5	66.4605	76	20.89366
6	61.5763	80	26.82473
7	59.3517	58	23.68760
8	58.6463	41	21.81289
9∗	79.0825	97	13.56441
Total	67.3414	677	22.45017

Table 2. ANOVA test on common final exam performance.

	SS	df	MS	F	*p*
Between groups	24,425.535	8	3,053.192	6.448	$< .00001$
Within groups	316,285.367	668	473.481		
Total	340,710.902	676			

guidance. The intentional backward-design of embedding "exam level" problems across the course to prepare students to master course-learning outcomes proved to be a successful practice amplified by the extra support offered in the flipped classroom [34].

The effects of aligning assessment and instruction in this manner were seen on exams, namely, the common final examination. We observed a gradual performance increase over the three major exams, with the final exam median being the highest among all three exam medians. Table 1 provides summative data on the common cumulative final examination taken by all Math 115 students in the Fall 2019 semester, where "Section 9"∗ represents the flipped version of Math 115.

Looking at the flipped classroom compared to the other eight sections, the mean exam score observed by the non-flipped eight sections was 65.3778, whereas the mean score observed by the flipped section was 79.0825. A one-way ANOVA on the mean common final exam results among all sections indicated a significant difference in means across sections $F(8, 668) = 6.448$, $p < .00001$, as shown in Table 2.

Post-hoc comparisons using the Tukey HSD test indicated that the mean score for the flipped class, section #9 ($M = 79.0825$, SD $= 13.56441$) was significantly different from the results obtained in seven other lecture sections. Table 3 summarizes the results of the Tukey HSD test.

In the spirit of reducing the D/F/W rate for the course, we also observed an approximately 15% D/F/W rate for our course, which was significantly lower than that of previous semesters. Approximately 56% of students earned a final grade of "B" or higher. This accomplished one of our P2C2 goals of increasing students' achievement in a gateway mathematics course and was consistent with research findings on improving students' achievement in STEM courses by using the flipped classroom approach [31]. Table 4 provides a summary of students' final grades.

Table 3. Tukey HSD post-hoc comparisons.

	Mean difference	Significance	95% CI Lower Bound	95% CI Upper Bound
Sec. 9 to Sec. 2	12.1601	$p = .0056$	2.1030	22.2172
Sec. 9 to Sec. 3	10.2060	$p = .0492$	0.0174	20.3946
Sec. 9 to Sec. 4	13.8617	$p = .0011$	3.5299	24.1935
Sec. 9 to Sec. 5	12.6220	$p = .0051$	2.2524	22.9916
Sec. 9 to Sec. 6	17.5062	$p < .0001$	7.2830	27.7294
Sec. 9 to Sec. 7	19.7308	$p < .0001$	8.4951	30.9665
Sec. 9 to Sec. 8	20.4362	$p < .0001$	7.8268	33.0456

Table 4. Final course grades in flipped Math 115.

Letter	A	B+	B	C+	C	D	F	W
No. of students	18	22	16	18	10	7	7	1

Students' perception of their performance was an item on the end-of-course evaluation. Sixty students responded to this anonymous evaluation. A supplemental question provided for this specific section was, "Do you think you would have received a lower, the same, or higher grade if you had been in a regular classroom?" Of the 42 students who responded to the question, 52% of students responded to this question with "higher," 31% responded with "the same," and 17% responded with "lower." Indeed, prior research [17] confirms that the increased flexibility and creativity in teaching that comes with a flipped classroom improves students' learning achievement and learning attitudes, which may have accounted for the observed results.

3.3. Building a Supportive Learning Community

Having near-peer ULAs allowed for more support to help keep students on task, answer questions, and offer program and college-level advice that sometimes instructors cannot. Students felt very comfortable asking ULAs for help, which positively affected the classroom climate and led to a true learning culture. Having a variety of support materials on the course learning management was also helpful; students were able to access technology applets, released exam questions for each lesson, homework scaffolds for when they were at home, and study outlines made by the instructor. The ULAs reinforced how vital these support tools were, which added another layer of credibility to the instructor's recommendations.

The combination of learning supports and classroom design was highly rated on the end-of-course evaluation. Table 5 shows the section-specific ($N = 60$) and course-wide (N unknown, but 650 students max) averages for three questions related to the classroom learning environment.

Based on this information, an overwhelming number of students seemed to agree and strongly agree that the flipped classroom instructional design in precalculus helped them learn a substantial amount of content. Having the ULAs in the classroom shifted teachers' and the students' attitudes in a positive way,

Table 5. Student instructional rating survey responses.

Question	Section	All Sections
"The instructional methods encouraged student learning."	4.74	4.01
"I learned a great deal in this course."	4.44	3.94
"I rate the overall quality of the course as:"	4.51	3.77

focused on evidence-based teaching and learning in an environment with constant support [25].

3.4. Promoting Open Communication

Reflection and goal-setting, as previously described, also allowed for more open dialogue between the instructor and the students and served as a vehicle for course feedback and actionable steps that could be taken to improve the learning experience. For example, the instructor was able to identify a handful of students early on who expressed dissatisfaction or frustration with the flipped classroom model in the exam wrapper early on and could intervene and help them move forward. For example, one student had trouble with time management outside of class, and another had a negative flipped classroom experience in high school. Having these pointed conversations with students and offering them feedback to improve their learning ability in a flipped classroom went a long way. Also, having feedback about the course structure allowed for student voice and minor revisions to the course as time went on.

Having two short quizzes during the in-person classes helped keep students in the class for the whole time and greatly minimized attendance issues often present in large lectures with no accountability system. On most evenings, at least 90% of students were present in class for its entirety. This is not the case for traditional lectures, where passive attendance is often encouraged.

Sample anonymous comments about course structure from the course evaluation also support the efficacy of open communication in the flipped classroom. Comments included:

- "I loved how the class was so hands-on. It was hard to fall behind in this class because *there was always someone coming up to me during class asking if I needed any help or if I was doing well*."
- "I loved the flipped classroom setting, and it made it so much easier for me to learn. I could do the notes at my own rate and I was responsible for it myself, but *at the same time I was able to get help in class about questions I was struggling with*."
- "I like that during lecture we have *opportunities to ask the Professor whatever questions we might have and that he is able to explain it in an understandable way*."

It should be noted that the students in the flipped section were all engineers. The results could have been impacted by the fact that these students were already heading toward a STEM-related field. However, most students who enter the School of Engineering begin with Calculus 1 or Calculus 2 for Engineers or even Multivariable Calculus; these students were placed into the lowest allowable mathematics course for the engineering program.

4. FUTURE IMPLEMENTATIONS

Future iterations of this course design will continue to draw on the design practices discussed in the MAA Instructional Practices Guide [1], including designing for equity, designing a flipped classroom that promotes reflective instruction, providing challenging mathematics problems, and aligning assessment to instruction through backward design. Many of the classroom practices used within the course design were thoroughly researched in the MAA Instructional Practices Guide [1], including fostering student engagement (collaboration/peer instruction), selecting appropriate tasks (engineering applications), formative and summative assessments of learning (including exam wrappers), and effective use of technology (video instruction, clicker quizzes, and real-time visual demonstrations). An improvement for future implementation of the course is to take advantage of platforms that track students' video-watching. Using these platforms ensures a greater degree of active learning on the students' part outside of the classroom, more accountability for learning over the "honor system" used in this implementation, and additional opportunities for students to engage with the content via embedded questions. Their responses to pre-class questions can also guide the start-of-class review more effectively and efficiently by allowing the instructor to highlight common misconceptions.

In reflecting on the very important roles of the ULAs, developing a pool of ULAs who are experienced in taking flipped courses such as this one will be important for staffing ULAs in this type of course in the future. These students will have a better frame of reference for what it is like to be students in a flipped classroom and can offer advice and strategies for students to be successful learners in the flipped classroom. The ULAs in this particular implementation of the course were very knowledgeable of active learning strategies and ways to increase cooperative learning opportunities but having ULAs who took a flipped course as this one would allow for an additional connection with the students. With an increasing number of classes moving to a fully online format, it might even be possible to repurpose some of the time ULAs spend in class to offer additional short sessions outside of class where they can coach students on how to learn mathematics in a flipped classroom.

In reflecting on the efficacy of the engineering applications, a future iteration of the course could involve a deviation from the coordinated quizzes in the lecture to place more value on the engineering applications. Replacing the time it takes to implement a start and end-of-class quiz with time spent on working on and submitting the engineering application could serve two useful purposes: (1) to hold students accountable for working through all activities in the flipped classroom and (2) to ensure students engage with the content at a higher level as expected in a flipped classroom. This also has the added benefit of more significant student investment in more challenging conceptual content since they will be earning some type of participation score for submitting their work. Relatedly, these group-worthy tasks rely on collaborative learning and peer instruction, as discussed in the MAA Instructional Practices Guide [1]. In the future, these tasks could form the basis for a newer kind of course assessment that is more project-based instead of traditional.

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